CHARACTERIZATION OF A POINT ON SURFACE BY GEOLOGIC UNITS NEIGHBOURING AROUND IT

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ABSTRACT

A concept of geologic units neighbouring around a point on surface and a function which assigns geologic units neighbouring around a point to the point, which is termed neighbourhood function, have been introduced to characterize a point on surface. The characterized points can be classified as a group of points which compose a specific geologic boundary surface. The algorithm for finding geologic units neighbouring around a point on surface is presented. Further, a method for classifying points on surface as a group of points which compose a specific geologic boundary surface is shown.

1. INTRODUCTION

Geologic boundary surfaces and geologic boundary lines are important elements which compose a three-dimensional geologic model. Shiono *et al*. (2004) have presented mathematical expression of geologic boundary surface and line based on the logical model of geologic structure by introducing the concept of a closure in topology. As another approach, a concept of geologic units neighbouring around a point on surface and a function which assigns geologic units neighbouring around a point to the point, which is termed neighbourhood function, are introduced to extract specific geologic boundary surface and line. If a point on surface is characterized by geologic units neighbouring around it, points on the surface can be classified as a group of points which compose a specific geologic boundary surface. An algorithm for finding geologic units neighbouring around a point on surface is presented. Further, a method for classifying points on surface as a group of points which compose specific geologic boundary surface and line is shown.

2. BASIC THEORY

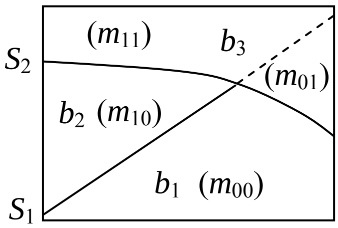
2.1 Logical model of geologic structure and geologic function

Let a 3-D subspace Ω be a survey area and suppose that the area Ω is composed of *n* geologic units that are disjoint:

$$b_1 \cup b_2 \cup \cdots \cup b_n = \Omega , \tag{1}$$

$$b_i \cap b_j = \phi \quad (i \neq j). \tag{2}$$

(a)



(b)

	S_1	S_2
b_1	0	0
b_2	1	0
b_3	*	1

(c)

minset	unit
m_{00}	b_1
m_{01}	b_3
m_{10}	b_2
m_{11}	b_3

Figure 1. Basic elements of a geologic model. (a) relation between geologic units and surfaces in geologic section, (b) logical model (1 ; S_i^+ , 0 ; S_i^- , * ; no specific relation with the surface.), (c) relational code table.

A geologic function g which assigns a unique geologic unit to every point in the 3-D space Ω has been introduced (Masumoto *et al*., 2004). The geologic function g is used to realize a 3-D geologic visualization in the GIS environment.

$$g : \Omega \to B, \quad \text{where } B = \{ b_1, b_2, \ldots, b_n\} \ . \tag{3}$$

Fundamentals of the geologic function g is explained using a simple geologic structure composed of three geologic units as shown in Figure 1(a). Three geologic units b_1, b_2 and b_3 are defined by two boundary surface S_1 and S_2 which divide Ω into two subspaces as follow;

$$b_1 = S_1^- \cap S_2^- , \quad b_2 = S_1^+ \cap S_2^- , \quad b_3 = S_2^+ , \tag{4}$$

where S_i^+ and S_i^- give subspaces that lie above and below the surface S_i, respectively. These equations can be expressed in a tabular form as shown in Figure 1(b). The above equations and table define the relation between geologic units and boundary surfaces. This logical relation is termed "the logical model of geologic structure" (Sakamoto *et al*., 1993).

As the geologic units b_1, b_2, …, b_n are defined by surfaces, they can be expressed in a "*minset standard form*" (Gill, 1976). The *minset* is a minimum subspace that is divided by the boundary surfaces S_1, S_2, …, S_p in the 3-D space Ω. Let $m_{d1d2\ldots dp}$ be a *minset* defined by;

$$m_{d1d2\ldots dp} = h_1(d_1) \cap h_2(d_2) \cap \ldots \cap h_p(d_p) \ , \tag{5}$$

$$\text{Where } h_i(d_i) = \begin{Bmatrix} S_i^+ ; d_i = 1 \\ S_i^- ; d_i = 0 \end{Bmatrix} .$$

In the case of Figure 1(a), four *minsets* can be defined as follows;

$$m_{00} = S_1^- \cap S_2^- , \quad m_{01} = S_1^- \cap S_2^+ , \quad m_{10} = S_1^+ \cap S_2^- , \quad m_{11} = S_1^+ \cap S_2^+ . \tag{6}$$

The *minset standard forms* can be derived for the geologic units as follows;

$$b_1 = m_{00} \ , \quad b_2 = m_{10} \ , \quad b_3 = m_{01} \cup m_{11} \ . \tag{7}$$

It is evident that each *minset* is included in only one of geologic units as shown below;

$$m_{00} \subset b_1 \ , \quad m_{01} \subset b_3 \ , \quad m_{10} \subset b_2 \ , \quad m_{11} \subset b_3 \ . \tag{8}$$

The relation between *minsets* and geologic units can be expressed by a function g_1 from a class of *minsets* M into B:

$$g_1 : M \to B \,. \tag{9}$$

The function g_1 can be represented by the relational code table shown in Figure 1(c).

Further, for a point $P(x, y, z)$ in a space Ω, a *minset* $m_{d1d2\ldots dp}$ can be assigned a value of $d_i = 1$ or $d_i = 0$ depending on whether $P(x, y, z)$ falls in S_i^+ or S_i^-, respectively. This correspondence between every point in Ω and *minsets* is expressed by a function g_2:

$$g_2 : \Omega \to M \,. \tag{10}$$

Consequently, a convolution of functions g_1: $M \to B$ and g_2: $\Omega \to M$ provides a rule to define the geologic unit that includes a given point $P(x, y, z)$:

$$g\,(x, y, z) = g_1\,(\,g_2\,(x, y, z))\,. \tag{11}$$

The function $g : \Omega \to B$ defines a rule to assign a unique geologic unit to every point in a 3-D space Ω.

2.2 Neighbourhood function

2.2.1 *Neighbourhood function of function g*

An open ball of radius ε centred at point X is called ε-neighbourhood of X and is denoted by $V(X, \varepsilon)$. A image of $V(X, \varepsilon)$ by the geologic function g which assigns a unique geologic unit to every point in the 3-D space Ω shows a set of geologic units in ε-neighbourhood of X (Figure 2):

$$g\,(V\,(X, \varepsilon)) = \{g(x) : x \in V\,(X, \varepsilon)\}\,. \tag{12}$$

As the radius ε becomes small, the number of geologic units in $V(X, \varepsilon)$ decreases. When $g(V(X, \varepsilon))$ becomes the same set for all the smaller radii ε than a certain value, $g\,(V\,(X, \varepsilon))$ is called "geologic units neighbouring around X" and is denoted by $\min g\,(V\,(X, \varepsilon))$.

A function G: $\Omega \to 2^B$ which assigns $\min g\,(V\,(X, \varepsilon))$ to X is called "neighbourhood function of function g" if $\min g\,(V\,(X, \varepsilon))$ exists for all points X in the 3-D space Ω (Figure 3).

2.2.2 *Neighbourhood function of function* g_2

The neighbourhood function can be expanded to various functions. The neighbourhood function of the function g_2 is defined as follows.

A image of $V(X, \varepsilon)$ by the function g_2 which assigns a unique *minset* to every point in the 3-D space Ω shows a set of *minsets* in ε-neighbourhood of X:

$$g_2\,(V\,(X, \varepsilon)) = \{g_2\,(x) : x \in V\,(X, \varepsilon)\}\,. \tag{13}$$

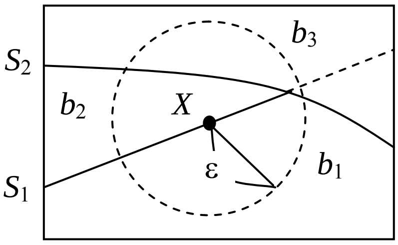


Figure 2. ε-neighbourhood of *X*.

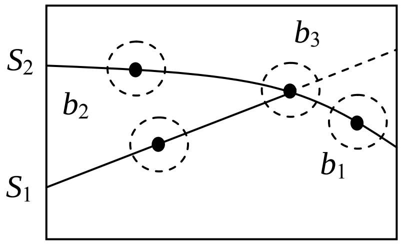


Figure 3. Neighbourhood function of function *g*.

Table 1. Relation between point *X* and surfaces.

	S_1	S_2
X	*	0

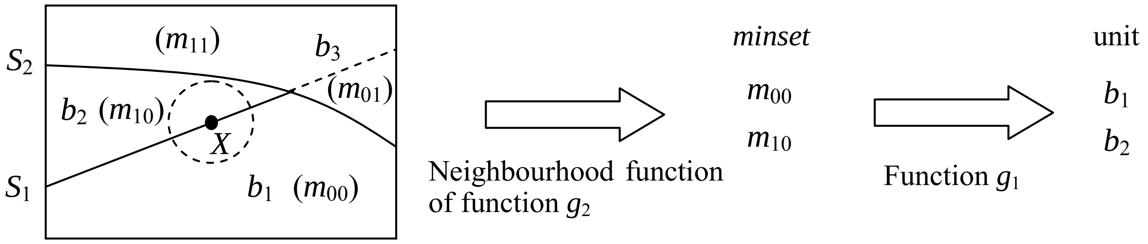


Figure 4. Geologic units neighbouring around a point *X*.

When g_2 ($V(X$, ε)) becomes the same set for all the smaller radii ε than a certain value, g_2 (V (X, ε)) is called "*minsets* neighbouring around X" and is denoted by min g_2 (V (X, ε)) .

A function $G_2 : \Omega \rightarrow 2^M$ which assigns min g_2 (V (X, ε)) to X is called "neighbourhood function of function g_2" if min g_2 (V (X, ε)) exists for all points X in the 3-D space Ω.

3. GEOLOGIC UNITS NEIGHBOURING AROUND A POINT

The geologic units neighbouring around a point on surface can be found using the neighbourhood function of function g_2 and the function g_1 (Figure 4). As an example, steps for finding geologic units neighbouring around the point X in Figure 4 are shown below.

Step 1; generate the relational code table from the logical model of geologic structure (Figure 1(c)). This means that the function g_1 is prepared.

Step 2; obtain the relation of the height between the point X and every boundary surface. If the point X is higher than a surface, the relation is expressed by "1". If lower then the relation is "0", if on the surface then the relation is "*". The relation between the point X and surfaces is shown in Table 1.

Step 3; find *minsets* neighbouring around the point X using the relation of the height between the point X and every boundary surface. The relation between a point and surfaces corresponds to subscript $d_1d_2...d_p$ of *minsets* neighbouring around the point. When the point X is higher than the surface S_i, the *minsets* neighbouring around the point X is also in upper side of the surface. Therefore, d_i is "1" when the relation between the point X and surface S_i is "1". Similarly, d_i is "0" when the relation between the point X and surface S_i is "0". When the point X is on the surface S_i, the *minsets* neighbouring around the point X exist in both the upper side and lower side of the surface. Therefore, d_i is both "1" and "0". According to the rule above, the *minsets* neighbouring around the point X in Figure 4 are m_{00} and m_{10}.

Step 4; derive geologic units from *minsets* neighbouring around the point X using the relational code table. In the case of this example, the geologic units neighbouring around the point X are b_1 and b_2.

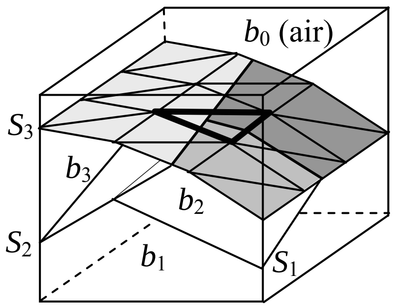


Figure 5. Geologic model.

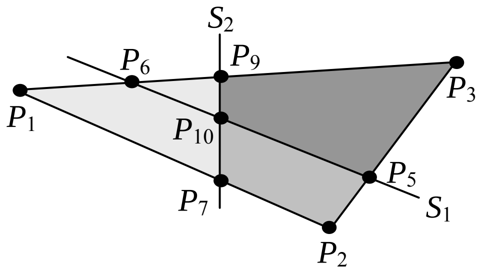


Figure 6. Triangular mesh.

Table 2. Logical model of geologic structure.

b_0	*	*	1
	S_1	S_2	S_3
b_1	0	0	0
b_2	1	0	0
b_3	*	1	0

Table 3. Relational code table.

minset	unit
m_{000}	b_1
m_{001}	b_0
m_{010}	b_3
m_{011}	b_0
m_{100}	b_2
m_{101}	b_0
m_{110}	b_3
m_{111}	b_0

Table 4. Geologic units neighbouring around a point on surface.

	Coordinates			Triangular sides			Intersection line between surface and triangle		Relation of height			Geologic units in the neighbourhood				
	x	y	z	P_1P_2	P_2P_3	P_1P_3	$S_1 \cap \Delta$	$S_2 \cap \Delta$	S_1	S_2	S_3	b_0	b_1	b_2	b_3	
P_1				1		1			1	1	*	1			1	
P_2				1	1				1	0	*	1		1		
P_3					1	1			0	0	*	1	1			
P_4				1			1									outside
P_5					1		1		*	0	*	1	1	1		
P_6						1	1		*	1	*	1			1	
P_7				1				1	1	*	*	1		1	1	
P_8					1			1								outside
P_9						1		1	0	*	*	1	1		1	
P_{10}							1	1	*	*	*	1	1	1	1	

4. CLASSIFICATION OF POINTS ON SURFACE

Using a geologic model of Figure 5, geologic units neighbouring around a point on surface are obtained and points on surface are classify as a group of points which compose a specific geologic boundary. Three boundary surfaces S_1, S_2 and S_3 are grid data and the logical model of geologic structure is given in Table 2.

First, the relational code table is prepared from the logical model of geologic structure (Table 3). Secondly, each grid cell is divided into two triangular meshes and following steps are implemented in each triangular mesh. As an example, the triangular mesh surrounded by a thick line in Figure 5 is used (Figure 6).

Step 1; assign a number to points on surface. P_1, P_2 and P_3 are assigned to each of triangular vertexes. P_4, P_5 and P_6 are given to each of points where three sides of the triangle and surface S_1 cross. Similarly, the intersection points between three sides of the triangle and surface S_2 are P_7, P_8 and P_9, respectively. P_{10} is assigned to the point where S_1, S_2 and triangle intersect.

Step 2; prepare a table to obtain geologic units neighbouring around the point (Table 4). "The relation between point and triangular sides" and "the relation between point and intersection line between surface and triangle" are the information which shows the position of the point. For example, if a point is on P_1P_2, the relation between the point and P_1P_2 is "1" and if a point is on the intersection line between S_1 and the triangular mesh, the relation between the point and $S_1 \cap \Delta$ is "1".

Step 3; calculate the coordinates of each point and find geologic units neighbouring around the point by the proposed algorithm. P_4 and P_8 are outside of the triangular mesh in the case of this example.

Step 4; classify the points on surface as a group of points which compose specific geologic boundary using the completed table. For example, the points which compose the geologic boundary surface between b_0 and b_1 are P_3, P_5, P_9 and P_{10}, which have b_0 and b_1 in the neighbourhood. The points which compose the geologic boundary line between b_1 and b_2 on the topographic surface S_3 are P_5 and P_{10}, which have b_0, b_1 and b_2 in the neighbourhood.

5. CONCLUSION

In the present work, a concept of geologic units neighbouring around a point on surface and a neighbourhood function which assigns geologic units neighbouring around a point to the point have been introduced to characterize a point on surface. The characterized points can be classified as a group of points which compose a specific geologic boundary. The algorithm for obtaining geologic units neighbouring around a point on surface was developed. Further, the points on surface were classified as a group of points which compose a specific geologic boundary by the algorithm. An algorithm for constructing geologic boundary surface and line from extracted groups needs to be developed to visualize a specific geologic boundary and utilize it for various analyses.

6 REFERENCES

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