OPTIMAL DETERMINATION OF GEOLOGIC SURFACES BASED ON FIELD OBSERVATION INCLUDING EQUALITY-INEQUALITY CONSTRAINTS AND SLOPE INFORMATION

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ABSTRACT

We present a gridding algorithm for optimal determination of geologic surfaces toward 3D geologic modeling. The algorithm, coded in a Fortran program Horizon2000 and in a Visual Basic program Terramod2001, is designed to determine the smoothest surface that satisfies a given set of input data that include inequality constraints and slope information as well as normal equality constraints based on the penalty function method. The optimal surface is provided as a solution that minimizes the augmented objective function : (smoothness)+(penalty)×(goodness of fit), where the smoothness of surface is evaluated in a form of numerical integration, the goodness of fit is in a form of residual mean of squares, and the penalty is a parameter that controls the balance between the smoothness and the goodness of fit. Calculations for several types of input data reveal that the algorithm provides a powerful mean to determine geological surfaces consistent with a variety of observational data.

1. INTRODUCTION

Many algorithms have been proposed for gridding of geological surfaces based on irregularly distributed observational data (e.g. Davis,1986; Jones et al.,1986). Most of all methods aimed at determination of reasonable surfaces $z = f(x, y)$ that satisfy observed values z_k at locations (x_k, y_k) $(k=1,\ldots, N)$ or $z_k = f(x_k, y_k)$. However, observation of strike-dip is important to know the attitude of the surface. The slope information should be used as constraints on the partial derivatives of the surface. Further, if a geologic unit that is stratigraphically upper than the surface is observed at an outcrop, we know that the surface lies below the outcrop. This type of information provides an inequality constraint for the surface. The purpose of this paper is to present an algorithm for determination of geologic surfaces using slope information and inequality constraints, which we have used for 3D geologic modeling.

2 AVAILABLE DATA IN THE FIELD SURVEY

There are many types of data available in the field survey. For the simplicity of explanation, we suppose a situation that we determine a surface of a geologic horizon $z = f(x, y)$ in a Cartesian coordinates with x-axis pointing towards the east, the y-axis pointing towards the north and the vertical z-axis. For an observation data point (x_p, y_p, z_p), there are three possibilities that we can be use observational data for the surface determination.

(1) Equality constraints: If the surface is exposed at a location (x_p, y_p, z_p), the location provides an equality constraint for the surface:

$$f(x_p, y_p) = z_p \tag{1}$$

(2) Inequality constraints: If a geologic unit that is stratigraphically upper than the surface is exposed at a location (x_p, y_p, z_p), the location provides an inequality constraint for the surface;

$$f(x_p, y_p) < z_p \, , \tag{2a}$$

and if a geologic unit that is lower than the surface is exposed at a location (x_p, y_p, z_p), the location provides an inequality constraint for the surface;

$$f(x_p, y_p) > z_p \, . \tag{2b}$$

In order to distinguish three type of constraints (1), (2a) and (2b), we introduce a parameter l_p defined by : $l_p = 0$ for a constraint (1), $l_p < 0$ for (2a), and $l_p > 0$ for (2b).

(3) Slope information: Strike and dip of the surface measured at an outcrop (x_q, y_q, z_q) provide constraints for the partial derivatives $f_x(x, y)$ with respect to x and $f_y(x, y)$ with respect to y:

$$f_x(x_q, y_q) = -\tan\theta_q \sin\phi_q, \tag{3a}$$

$$f_y(x_q, y_q) = -\tan\theta_q \cos\phi_q, \tag{3b}$$

where ϕ_q is the trend of the maximum slope and θ_q is the dip angle.

3 ALGORITHM

There may exist many feasible solutions that satisfy all observation (1), (2a), (2b), (3a), and (3b). Assuming that the geologic surface must be the smoothest one among the feasible solutions, we consider the surface determination as an optimization problem:

Find an optimal solution $f(x, y)$ that minimizes

$$J(f) = m_1 \iint f_x(x, y)^2 + f_y(x, y)^2 \, dx \, dy$$
$$+ m_2 \iint f_{xx}(x, y)^2 + 2 f_{xy}(x, y)^2 + f_{yy}(x, y)^2 \, dx \, dy \tag{4}$$

subject to

$$f(x_p, y_p) - z_p = 0 \quad ; \text{if } l_p = 0,$$
$$f(x_p, y_p) - z_p < 0 \quad ; \text{if } l_p < 0, \quad (p = 1, \ldots, P)$$
$$f(x_p, y_p) - z_p > 0 \quad ; \text{if } l_p > 0,$$

and

$$f_x\,(x_q, y_q) + \tan\theta_q\ \sin\phi_q = 0$$

$$f_y\,(x_q, y_q) + \tan\theta_q\ \cos\phi_q = 0 \qquad (q = 1, \ldots, Q).$$

In order to find the numerical solution of the problem by the penalty function method, we evaluate the functional $J(f)$ by a numerical integration using all values $f_{11}, f_{12}, \ldots, f_{NxNy}$ at $Nx \times Ny$ nodes of a rectangular grid and approximate a function $f\,(x, y)$ by a surface of second degree using values at six neighboring nodes.

3.1 Normal Gridding

For a set of observation (1), we introduce an augmented objective function:

$$Q(f\,;\alpha) = J(f) + \alpha\, R_{\mathrm{H}}(f) \tag{5}$$

where $J(f)$ gives the smoothness of surface evaluated in a form of numerical integration, $R_{\mathrm{H}}(f)$ gives the goodness of fit in a form of residual mean of squares,

$$R_{\mathrm{H}}(f) = \Sigma\,(f(x_p, y_p) - z_p)^2 / n_{\mathrm{H}}\,, \tag{6}$$

α is a parameter called a penalty that controls the balance between the smoothness, and the goodness of fit and n_{H} is a number of data. As all terms in $Q(f\,;\alpha)$ are represented in a quadratic form of values at grid nodes, the optimal solution $\boldsymbol{f} = (f_{11}, f_{12}, \ldots, f_{NxNy})$ is given by a simultaneous equation

$$\mathrm{A}\,\boldsymbol{f} = \boldsymbol{b} \tag{7}$$

derived from $\partial Q / \partial f_{ij} = 0$ ($i = 1, \ldots, Nx$; $j = 1, \ldots, Ny$). Figure 1 gives examples using data modified from TABLE B.3 in Johns et al.(1986) and TABLE 5.11 in Davis(1986).

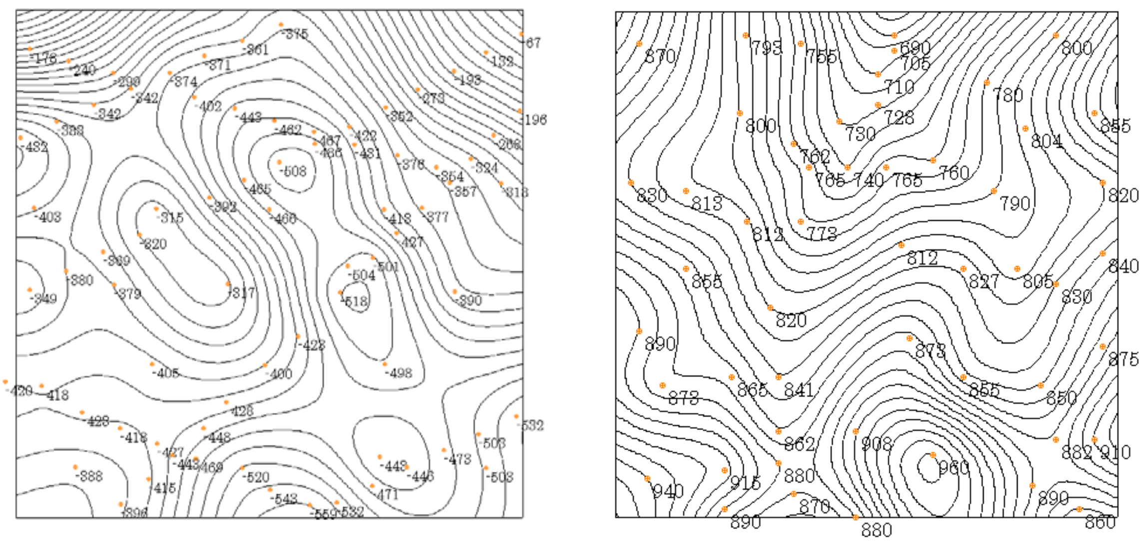


Figure 1. Example (1). Left: data modified from TABLE B.3 in Johns et al.(1986). Right : data modified from TABLE 5.11 in Davis(1986).

3.2 Slope Information

When n_D sets of strike and dip data are available in addition to the normal data (1), the augmented objective function is modified as follows:

$$Q(f\,;\alpha) = J(f) + \alpha\, R_H(f) + \beta\, R_D(f), \tag{8}$$

where

$$R_D(f) = \Sigma\ \{(f_x\,(x_q, y_q) + \tan\theta_q \sin\phi_q)^2 + (f_y\,(x_q, y_q) + \tan\theta_q \cos\phi_q,)^2\} / 2\, n_D. \tag{9}$$

and β is a penalty that controls the balance between $R_H(f)$ and $R_D(f)$. Figure 2 give an example using observational data at nine outcrops. All outcrops are located at 0 m and the surface dips at 45° eastward or westward.

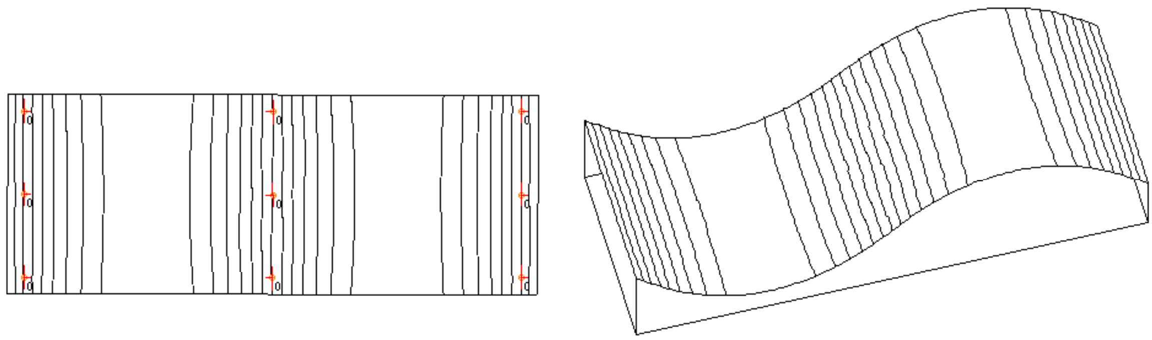


Figure 2. Example (2) .

3.3 Inequality Constraints

When inequality constraints (2a) and (2b) are obtained in addition to equality ones (1), the residual or the difference between the surface $f(x_p, y_p)$ and observation z_p is given by

$$\varepsilon_p = \begin{cases} f(x_p, y_p) - z_p & ;\text{ if } l_p = 0 \\ \max\ \{f(x_p, y_p) - z_p\,, 0\} & ;\text{ if } l_p < 0 \\ \min\ \{f(x_p, y_p) - z_p\,, 0\} & ;\text{ if } l_p > 0 \end{cases} \tag{10}$$

Therefore, $R_H(f)$ in the augmented objective function (5) and (8) should be evaluated by

$$R_H(f) = \Sigma\ \varepsilon_p^{\ 2} / n_H \tag{11}$$

where n_H is a number of data that *do not* satisfy constraints (1), (2a) and (2b). It is noted that the residual ε_p can be evaluated after a surface is determined. We apply the exterior penalty function method (Zangwill,1967) to solve the problem. The method requires us the iterative calculations with increasing series of penalties α_1, α_2, …, α_T to find a surface consistent with inequality constraints as follows:

(1) Suppose the t th solution $\boldsymbol{f}^{(t)}$ is given,

(2) Give an equality constraint $f(x_p, y_p) = z_p$ for the inequality constraint that surface $\boldsymbol{f}^{(t)}$ does not satisfy or $\varepsilon_p \neq 0$,

(3) Construct the augmented objective function $Q(f\,;\,\alpha_{t+1})$ to find the (t + 1) th solution $\boldsymbol{f}^{(t+1)}$.

Figure 3 gives an example in which all types of constraints are included.

(1) location of outcrops

p	x_p	y_p	z_p	l_p
1	7	16	20	0
2	19	80	37	1
3	23	43	42	0
4	87	23	45	-1
5	84	85	55	1

(2) slope information

q	x_p	y_p	z_p	ϕ_p	θ_p
1	13	82	32	285	40
2	28	35	45	220	30
3	34	8	35	200	40
4	47	83	48	285	30
5	77	93	50	300	20
6	64	59	65	220	20
7	72	14	39	170	40
8	90	47	45	150	30

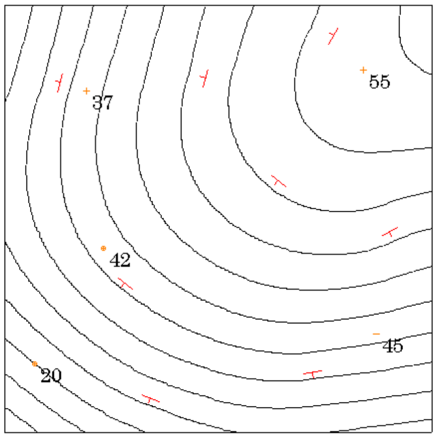
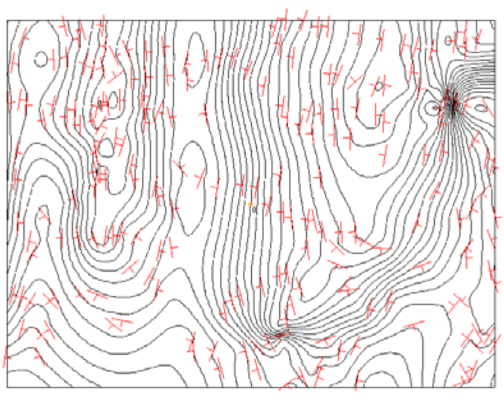


Figure 3. Example (3).

4 APPLICATIONS

The algorithm is coded in a Fortran program Horizon2000 (Shiono et al., 2001) and in a Visual Basic program Terramod2001 (Sakamoto et al., 2001). The former is available at a download site of Japan Society of Geoinformaics (http: // www.jsgi.org.jp/).

The program is useful for determination of geologic surfaces. One example is the fitting of a surface to strike and dip data. Figure 4 shows a surface determined only by strike and dip data digitized from a geologic map. It is easy to see an outline of folding structure and local variation of strike and dip.



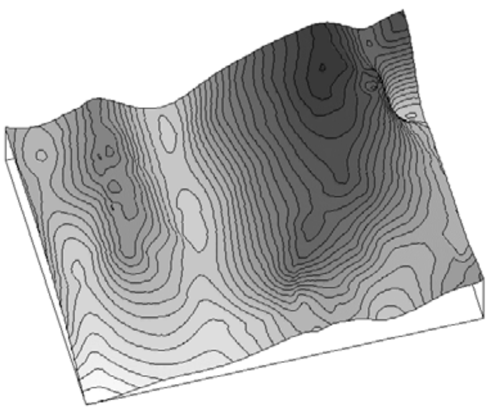


Figure 4. Surface determined by strike and dip data digitized from a geologic map.

Another important application is a efficient generation of DEM (Digital Elevation Model) from a topographic map (Noumi et al., 1999; Shiono et al., 2001). An elevation $f(x_p, y_p)$ at a point (x_p, y_p) in a space between two successive contour lines h and H must be

$$h < (x_p, y_p) < H.$$

Based on this idea, we can generate quickly a DEM by assigning the inequality constraints to each point in a space between contour lines after scanning a topographic map. Figure 5 shows an example.

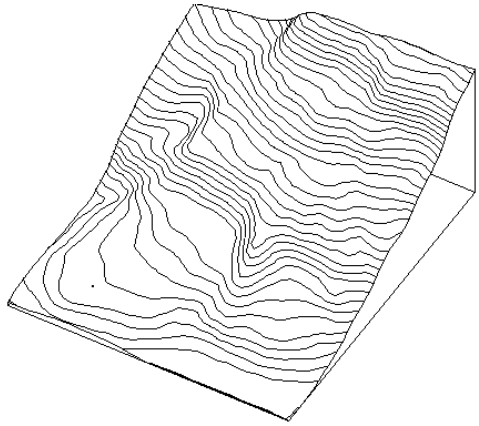


Figure 5. 3D visualization of DEM generated from a topographic map.

5 CONCLUSION

We present a gridding algorithm for optimal determination of geologic surfaces towards 3D geologic modeling. The algorithm is designed to determine the smoothest surface that satisfies a given set of observational data that include inequality constraints and slope information as well as normal equality constraints based on the penalty function method. The algorithm is coded in a Fortran program Horizon2000 and in a Visual Basic program Terramod2001. The calculations for several types of input data reveal that the algorithm provides a powerful tool to determine geological surfaces consistent with a variety of observational data.

6 REFERENCES

Davis, J. C., 1986. *Statistics and Data Analysis in Geology* (2nd Edition). John Wiley & Sons, New York.

Jones, T. A., Hamilton, D. E., and Johnson, C. R., 1986. *Contouring Geologic Surfaces with the Computer*. Van Nostrand Reinhold Company, New York.

Noumi, Y., Shiono, K., Masumoto, S., and Raghavan, V., 1999. Generation of DEM from the topographic maps – Utilization of inter-contour information. *Geoinformatics*, Vol.10, 235-246.

Sakamoto, M., Shiono, K., and Masumoto, S., 2001. Terramod2001: A software for gridding and modelling of geologic surface. *Geoinformatics,* Vol.12, 112-113.

Shiono, K., Noumi, Y., Masumoto, S., and Sakamoto, M., 2001. Horizon2000: Revised program for optimal determination of geologic surfaces based on field observation including equality-inequality constraints and slope information. *Geoinformatics*, Vol.12, 229-249.

Zangwill, W. I., 1967. Non-linear programming via penalty function. *Management Science*, Vol.13, 344-358.